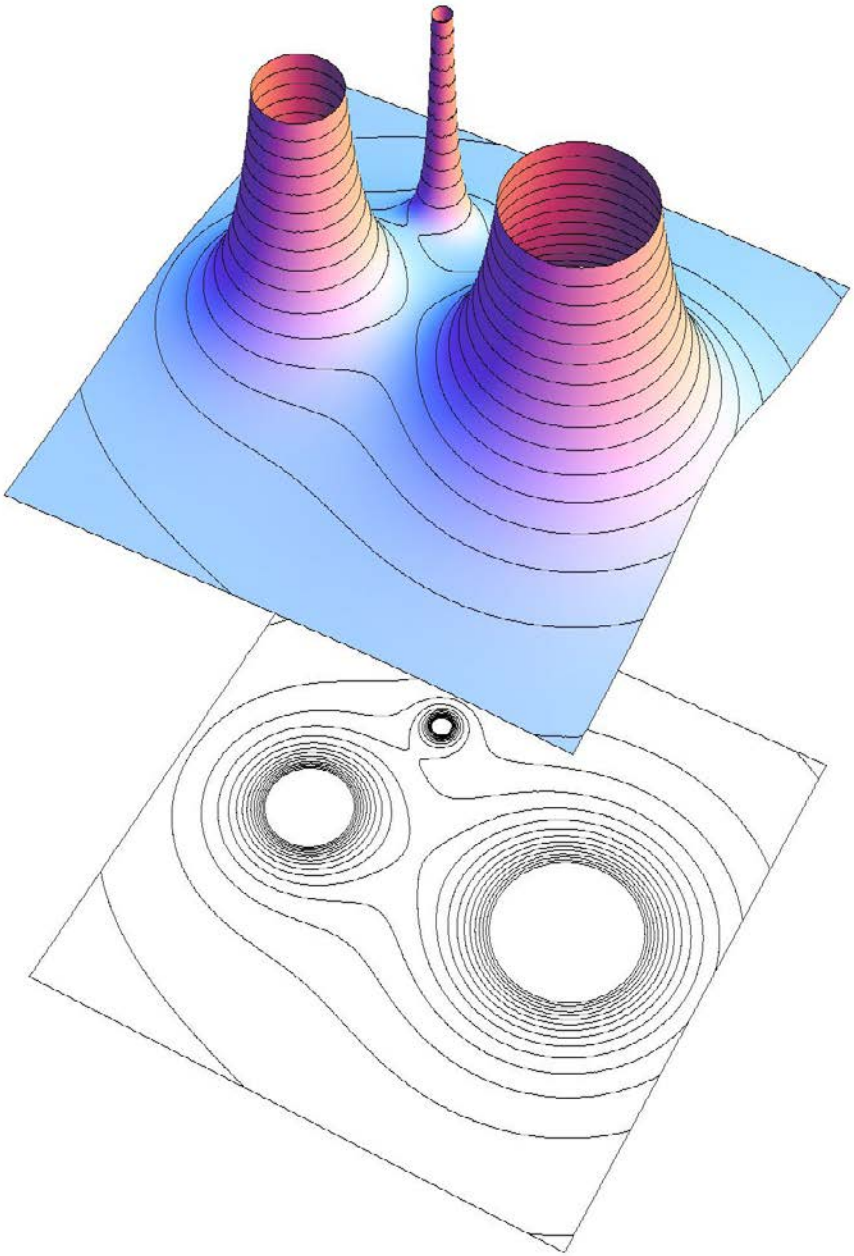


National Institute for Public Health
and the Environment
Ministry of Health, Welfare and Sport

NORM bulk and external radiation:

a proof of the sum rule for point sources at site boundaries

C.P. Tanzi and H.J. de Blank*

*FOM Institute DIFFER - Dutch Institute for Fundamental Energy Research, Association EURATOM-FOM, Nieuwegein, the Netherlands

Dose outside < sum of permitted limits at site boundaries

- Why?

1 mSv/y limit for members of the public: EURATOM Bss

96/29 Art.13
2013/59 Art.3

"No one will be exposed to more than 1 mSv/y"

- How to guarantee?

in Dutch legislation highest permitted dose limit 100 microSv/y
with the assumption of at most 10 sites

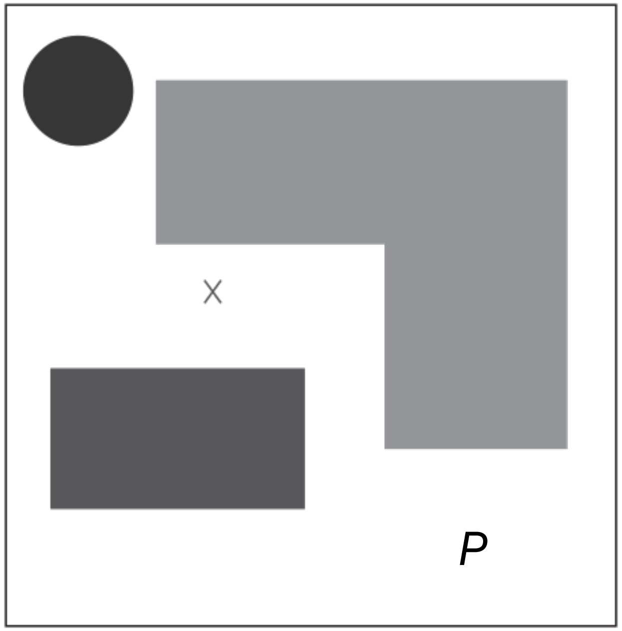
- Does it work? May it not be that... perhaps...

a surprising combination of sources...
and what about absorption? ... what about buildup?

Maximum dose at any point in the public domain cannot exceed the sum of the permitted doses

for point sources

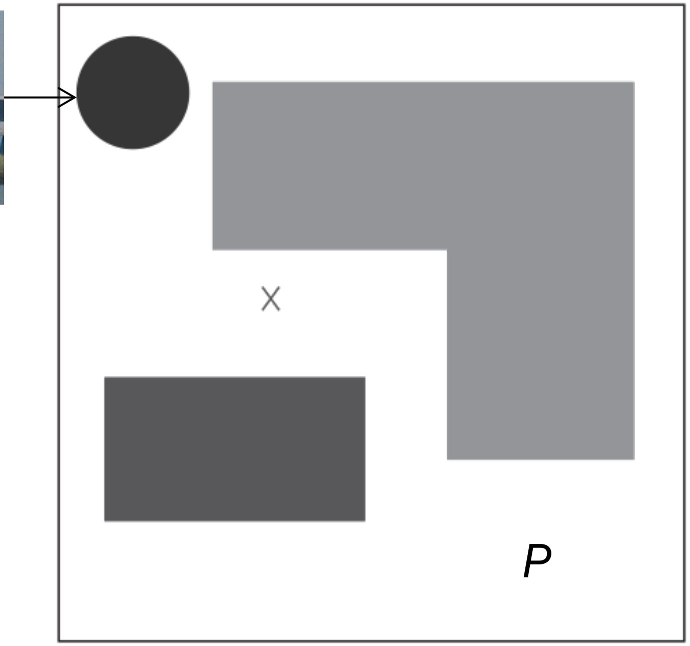
- for all locations outside the site perimeters
- for all possible site shapes and relative positions
- for all possible distributions of sources within the site perimeter



Maximum dose at any point in the public domain cannot exceed the sum of the permitted doses

for point sources

- for all locations outside the site perimeters
- for all possible site shapes and relative positions
- for all possible distributions of sources within the site perimeter

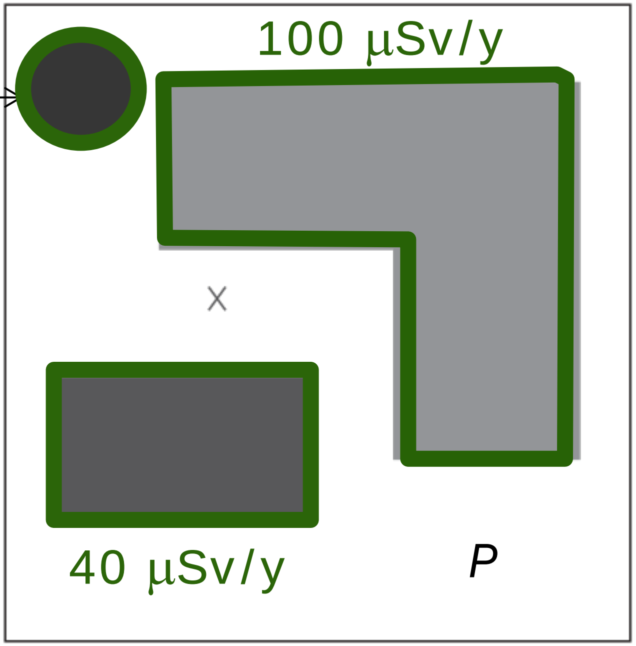


Maximum dose at any point in the public domain cannot exceed the sum of the permitted doses

for point sources

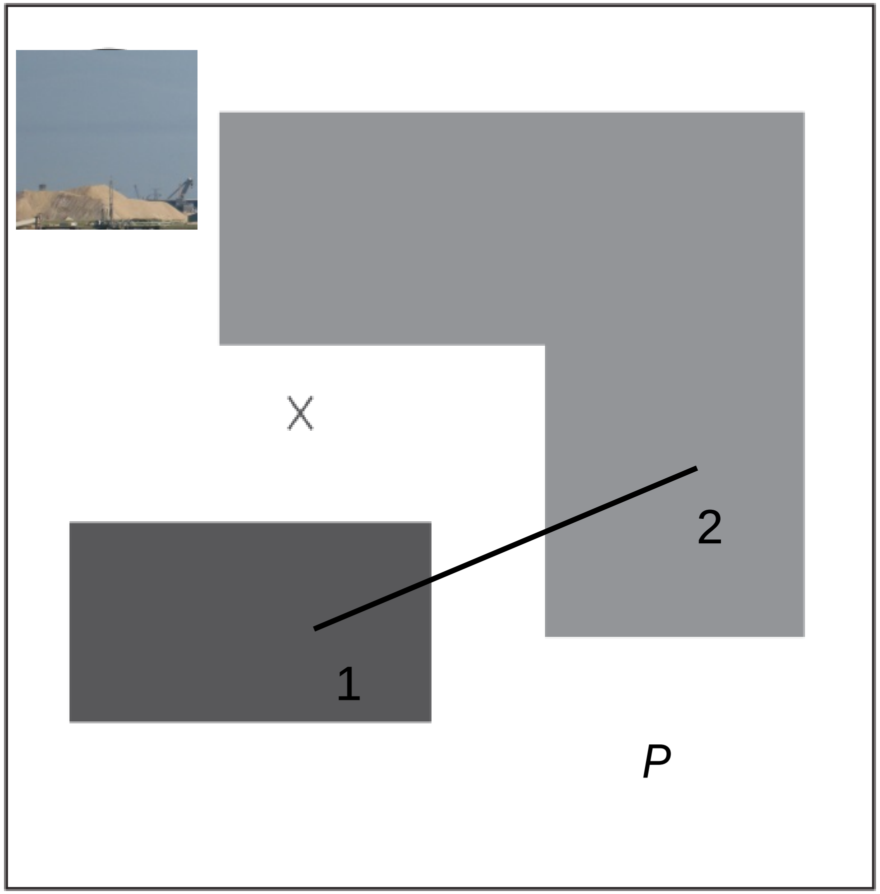
- for all locations outside the site perimeters
- for all possible site shapes and relative positions
- for all possible distributions of sources within the site perimeter

≤ 300 μSv/y

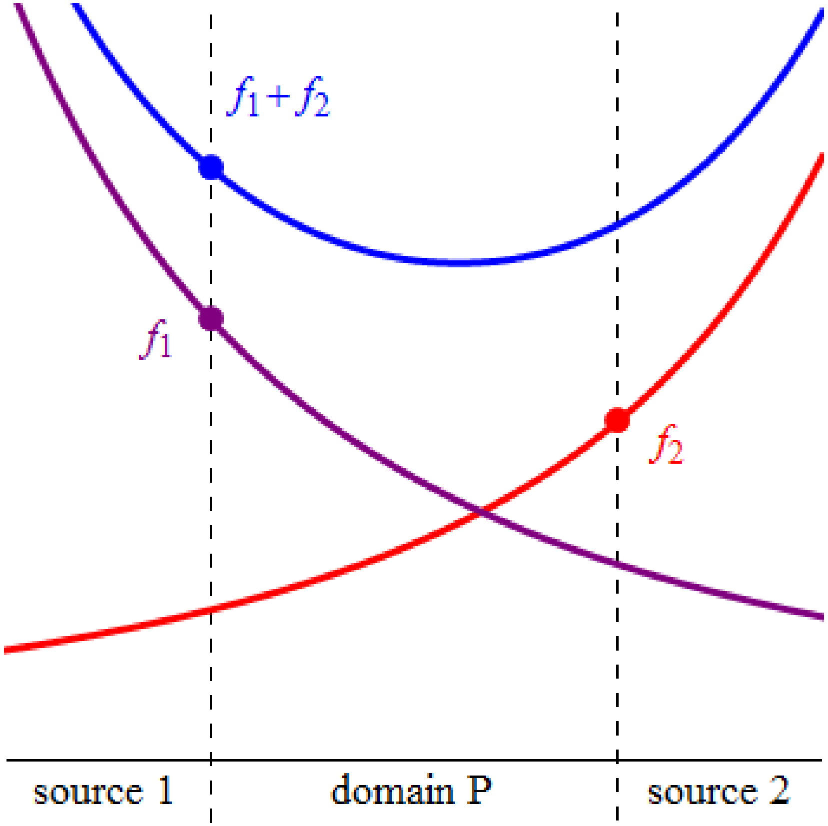


x ≤ 440 μSv/y everywhere outside the boundaries/fences

simple example: a 1-D cross-section



The radiation field $f(x+y)$ on the public domain *P* has its maximum value on the domain boundary ∂*P*



for a point source

f_1 and f_2 have positive Laplacian (second derivative)

→ the sum as well

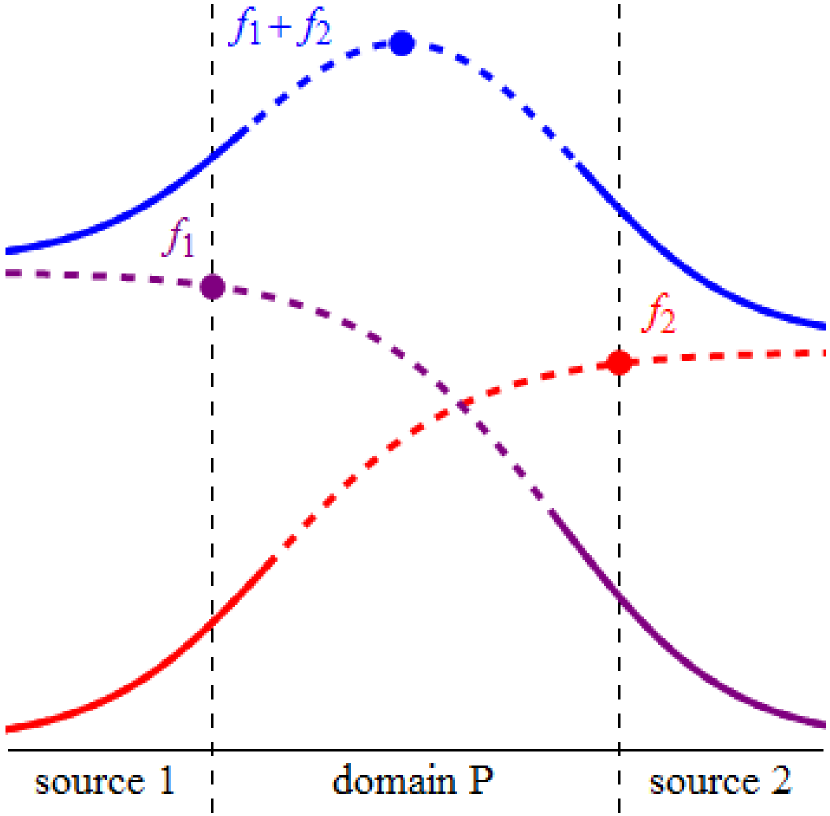
→ on the domain P,
the maximum of f_1 + f_2
lies on the boundary

Counterxample:

the functions f_1, f_2 have NEGATIVE second derivative

$f_1 + f_2$ also has NEGATIVE second derivative

→ on the domain P,
the maximum of $f_1 + f_2$
lies INSIDE the boundary



Maximum dose at any point in the public domain cannot exceed the sum of the permitted doses

for point sources

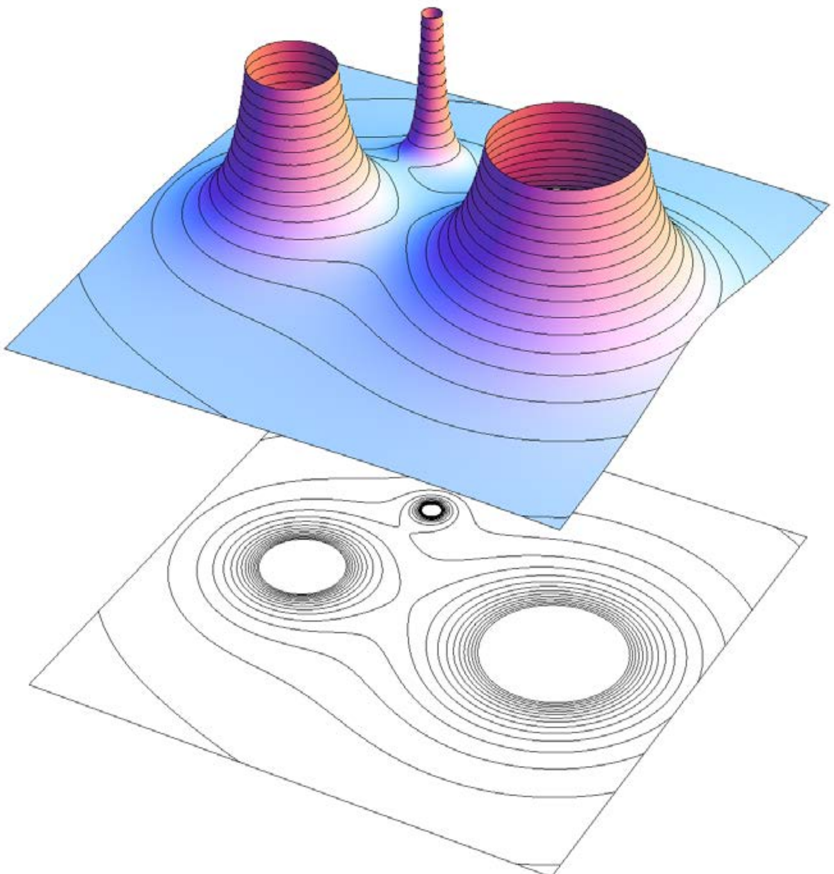
- for all locations outside the site perimeters
- for all possible site shapes and relative positions
- for all possible distributions of sources

the Laplacian differential operator
(divergence of the gradient, $\Delta = \nabla^2 = \nabla \cdot \nabla$)
possess the maximum property
(it has its largest value on the boundaries of its domain of definition)

Point source : $f(r) = \frac{c}{r^2} \rightarrow \Delta f(r) = \frac{4c}{r^4}$

→ for any distribution of sources, the Laplacian is positive

Laplacian: $\Delta f(r) = \frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr}$



Contour levels of effective dose caused by three arbitrary points.

All closed contours enclose a source!!

This guarantees that there is not a location where a local maximum occurs.

Radiation may lose energy before it is absorbed

$$f(r) = \frac{c}{r^2}\exp\left(-\int_0^r \frac{dx}{\lambda(x)}\right)$$ $-\,\lambda$ attenuation length

- **absorption: λ=constant,** $f(r) = \frac{c}{r^2}e^{-r/\lambda}$

$\rightarrow \Delta f > 0$ -------->  theorem applies

- **moderation**

if $\frac{d}{dr}(\lambda(r)\,r^7) > 0$ -------->  theorem applies

- **scattering and build-up**

integrate over all possible scattering positions

$$f_2(\boldsymbol{p}) = \int d^3\boldsymbol{p}'\, f_1(|\boldsymbol{p}'|)\, f_1(|\boldsymbol{p}-\boldsymbol{p}'|)$$

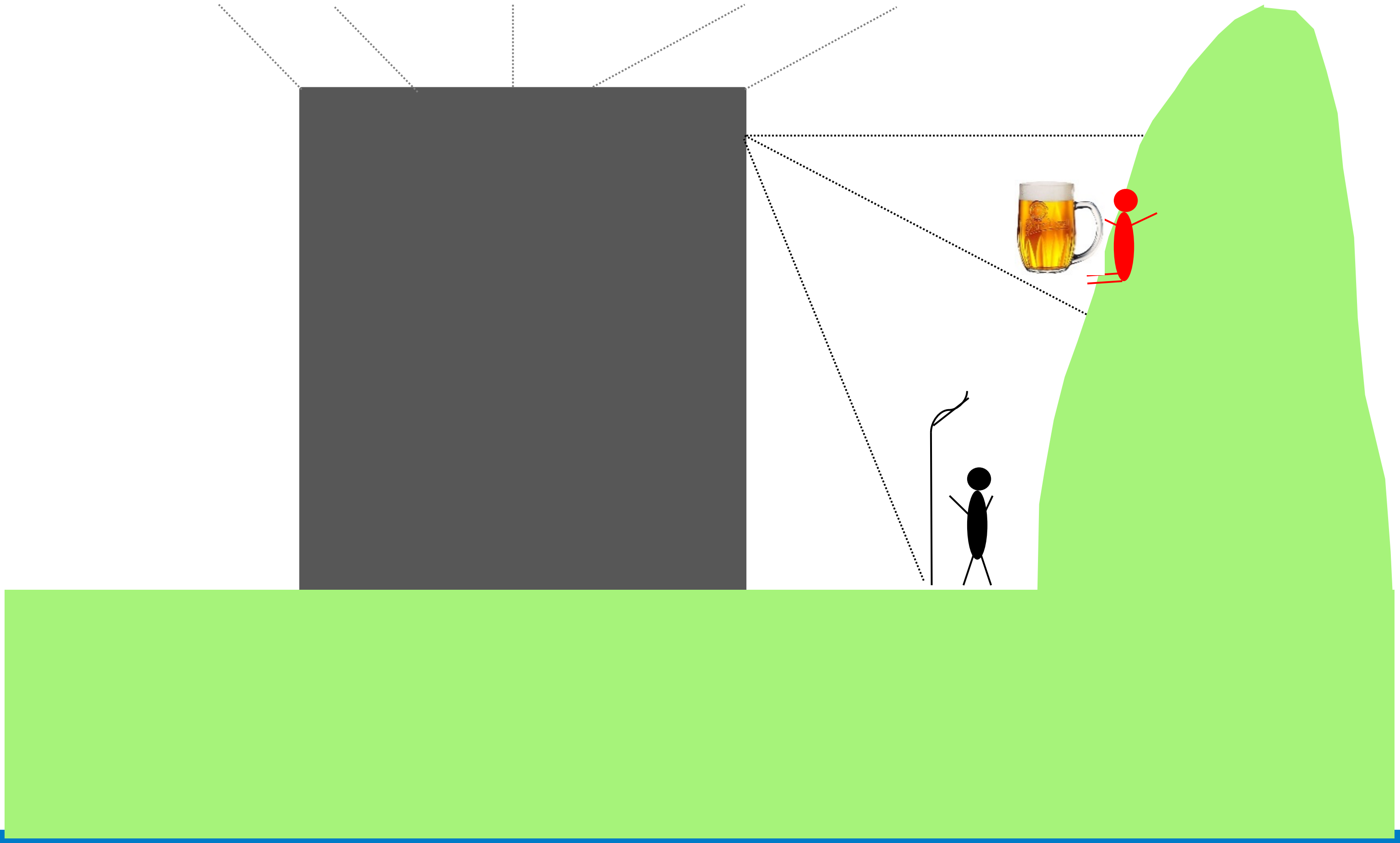
$$\Delta_3 = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} = \Delta + \frac{1}{r}\frac{d}{dr}$$

$$\Delta f_2(\boldsymbol{p}) > \Delta_3 f_2(\boldsymbol{p}) = \int d^3\boldsymbol{p}'\, f_1(|\boldsymbol{p}'|)\, \Delta_3 f_1(|\boldsymbol{p}-\boldsymbol{p}'|)$$

-------->  theorem applies

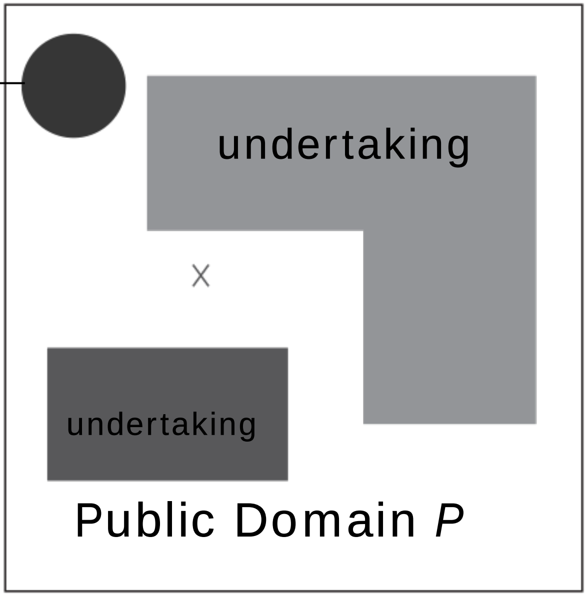
WARNING!!!

it does not work for altitude differences or skyshine



Conclusion

When set dose limits at site perimeters of undertakings are enforced, there can be no peculiar spatial arrangement of sources of radiation which leads to a combination where the dose might be higher than the simple sum of the dose given by the individual radiation fields*.

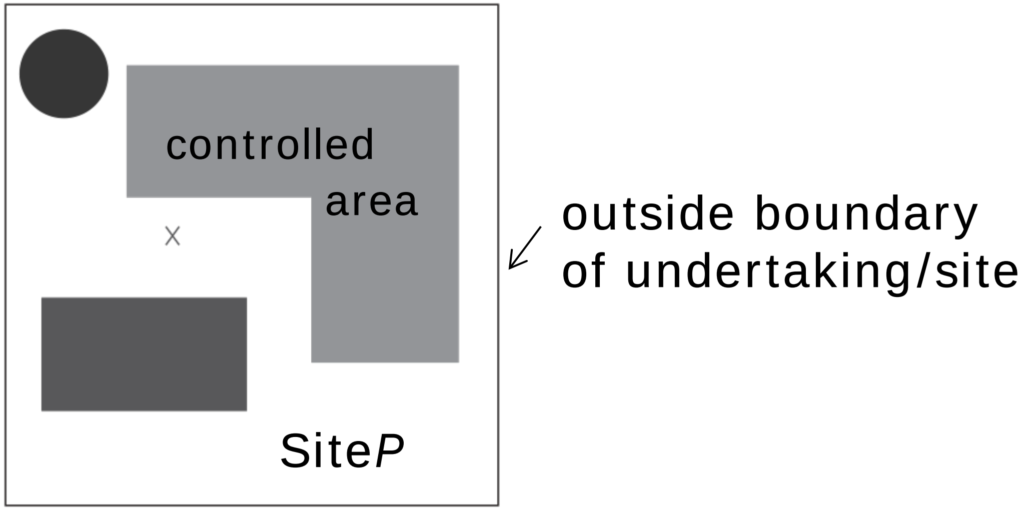


\+ outlook Bss

* with the exclusion of complex terrain

dose constraints consistent with dose limits (Euratom Directive 2013/59)

If dose constraints for activities within an undertaking are established, and the simple sum of the dose given by the individual radiation fields does not exceed the dose limit, consistency follows from this proof*.



* with the exclusion of complex terrain

extra slides

Attenuation length λ: $f(r) = \frac{c}{r^2} e^{-r/\lambda}$

…absorption…

$\Delta f > 0$ ->  theorem applies

…moderation…

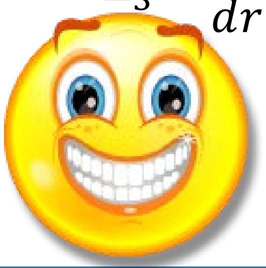
if $\frac{d}{dr}(\lambda r^7) > 0$ ->  theorem applies

…scattering and build-up…

secondary radiation: integrate over all possible scattering positions

$$f_2(\boldsymbol{p}) = \int d^3\boldsymbol{p}'\, f_1(|\boldsymbol{p}'|)\, f_1(|\boldsymbol{p}-\boldsymbol{p}'|)$$

$$\Delta_3 = \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} = \Delta + \frac{1}{r}\frac{d}{dr}$$

$\Delta f_2(\boldsymbol{p}) > \Delta_3 f_2(\boldsymbol{p}) = \int d^3\boldsymbol{p}'\, f_1(|\boldsymbol{p}'|)\, \Delta_3 f_1(|\boldsymbol{p}-\boldsymbol{p}'|)$ ->  theorem applies